1/2

77+10 $\boxed{87}$

18+15+12+10 +10+12

1. a. $a = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$

De lengte van a is:

$\|a\|^2 = (4)^2 + (0)^2 + (3)^2 = 16 + 9 = 25$

$\|a\| = \sqrt{25} = 5$

de vector met lengte 1 in de richting van a is:

3

$\frac{1}{5} \cdot a = \frac{1}{5} \cdot \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix}$

b. met $\alpha = 0$ $\quad b = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$ $\quad c = \begin{pmatrix} 3\sqrt{3} \\ 3 \\ 0 \end{pmatrix}$

De hoek ϑ tussen b en c kan worden berekend met:

5

$b \cdot c = \|b\| \|c\| \cdot \cos\vartheta$

$9\sqrt{3} = 3 \cdot 6 \cdot \cos\vartheta$

$\frac{1}{2}\sqrt{3} = \cos\vartheta$

$\vartheta = \cos^{-1}(\frac{1}{2}\sqrt{3})$

$\vartheta = \frac{1}{6}\pi \lor \vartheta = -\frac{1}{6}\pi$

$\|b\|^2 = (3)^2 + (0)^2 + (0)^2 = 9$

$\|b\| = \sqrt{9} = 3$

$\|c\|^2 = (3\sqrt{3})^2 + (3)^2 + (0)^2 = 27 + 9 = 36$

$\|c\| = \sqrt{36} = 6$.

c. a en c staan loodrecht op elkaar voor: $\vartheta = \frac{1}{2}\pi \lor \vartheta = -\frac{1}{2}\pi$

$\cos(\frac{1}{2}\pi) = 0$ en $\cos(-\frac{1}{2}\pi) = 0$

$a \cdot c = \|a\| \|c\| \cdot 0$.

4

$a \cdot c = 0$

$\begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3\sqrt{3} \\ 3 \\ \alpha \end{pmatrix} = 0$

$12\sqrt{3} + 0 + 3\alpha = 0$

$3\alpha = -12\sqrt{3}$

$\alpha = -4\sqrt{3}$.

Dus voor $\alpha = -4\sqrt{3}$ staan a en c loodrecht op elkaar.

d. a, b en c zijn lineair onafhankelijk als er voor de vergelijking:

$x_1 a + x_2 b + x_3 c = 0$

geen andere oplossing is dan $x_1 = x_2 = x_3 = 0$

6

$$x_1\begin{pmatrix}4\\0\\3\end{pmatrix} + x_2\begin{pmatrix}3\\0\\0\end{pmatrix} + x_3\begin{pmatrix}3\sqrt{3}\\3\\\alpha\end{pmatrix} = 0$$

Hieruit volgt:

$$\begin{aligned} 4x_1 + 3x_2 + 3\sqrt{3}x_3 &= 0 \\ 3x_3 &= 0 \\ 3x_1 + \alpha x_3 &= 0 \end{aligned}$$

In matrix vorm is dit:

$$\left(\begin{array}{ccc|c} 4 & 3 & 3\sqrt{3} & 0 \\ 0 & 0 & 3 & 0 \\ 3 & 0 & \alpha & 0 \end{array}\right)$$

$Rij_1 \to Rij_1$
$Rij_2 \to Rij_2 \cdot \frac{1}{3}$
$Rij_3 \to Rij_3$

$$\left(\begin{array}{ccc|c} 4 & 3 & 3\sqrt{3} & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & \alpha & 0 \end{array}\right)$$

$R_1 \to Rij_1 - 3\sqrt{3}\, Rij_2$
$Rij_2 \to Rij_2$
$Rij_3 \to (Rij_3 - \alpha Rij_2) \cdot \frac{1}{3}$

$$\left(\begin{array}{ccc|c} 4 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$$

$Rij_1 \to (Rij_1 - 4Rij_3) \cdot \frac{1}{3}$
$Rij_2 \to Rij_2$
$Rij_3 \to Rij_3$

$$\left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right)$$

oftewel:

$x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$

Dus a, b en c zijn lineair onafhankelijk voor elke α.

2a. met $\alpha = 2$ en $\beta = 1$:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 3 & 1 & 0 \\ 4 & 4 & 4 \end{pmatrix} \quad b = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$Ax = b$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 3 & 1 & 0 \\ 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

Hieruit volgt:

$$\begin{aligned} x_2 + 2x_3 &= 1 \\ 3x_1 + x_2 &= 2 \\ 4x_1 + 4x_2 + 4x_3 &= 0. \end{aligned}$$

In matrix vorm komt dit neer op:

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 3 & 1 & 0 & 2 \\ 4 & 4 & 4 & 0 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \\ Rij_2 \to Rij_2 - \frac{3}{4} Rij_3 \\ Rij_3 \to Rij_3 \cdot \frac{1}{4} \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 0 & -2 & -3 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \\ Rij_2 \to Rij_2 + 2 \cdot Rij_1 \\ Rij_3 \to Rij_3 - Rij_1 \end{array}$$

7

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & -1 & -1 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 - 2 Rij_2 \\ Rij_2 \to Rij_2 \\ Rij_3 \to Rij_3 + Rij_2. \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 3 \end{array}\right)$$ Hieruit volgt.

$x_1 = 3$, $x_2 = -7$, $x_3 = 4$.

Oftewel

$$x = \begin{pmatrix} 3 \\ -7 \\ 4 \end{pmatrix}$$

2b.

$$A = \begin{pmatrix} 0 & 1 & \alpha \\ 3 & 1 & 0 \\ 4 & 4 & 4 \end{pmatrix} \qquad b = \begin{pmatrix} \beta \\ 2 \\ 0 \end{pmatrix}$$

$Ax = b$

$$\left(\begin{array}{ccc|c} 0 & 1 & \alpha & \beta \\ 3 & 1 & 0 & 2 \\ 4 & 4 & 4 & 0 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \\ Rij_2 \to Rij_2 \\ Rij_3 \to Rij_3 \cdot \frac{1}{4} \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & \alpha & \beta \\ 3 & 1 & 0 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \\ Rij_2 \to Rij_2 - 3Rij_3 \\ Rij_3 \to Rij_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & \alpha & \beta \\ 0 & -2 & -3 & 2 \\ 1 & 1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \\ Rij_2 \to Rij_2 \cdot -\frac{1}{2} \\ Rij_3 \to Rij_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & \alpha & \beta \\ 0 & 1 & 1\frac{1}{2} & -1 \\ 1 & 1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 - Rij_2 \\ Rij_2 \to Rij_2 \\ Rij_3 \to Rij_3 - Rij_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 0 & \alpha - 1\frac{1}{2} & \beta + 1 \\ 0 & 1 & 1\frac{1}{2} & -1 \\ 1 & 0 & -\frac{1}{2} & 1 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \cdot \frac{1}{\alpha - 1\frac{1}{2}} \\ Rij_2 \to Rij_2 \\ Rij_3 \to Rij_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & \frac{\beta+1}{\alpha - 1\frac{1}{2}} \\ 0 & 1 & 1\frac{1}{2} & -1 \\ 1 & 0 & -\frac{1}{2} & 1 \end{array}\right) \quad \begin{array}{l} Rij_1 \to Rij_1 \\ Rij_2 \to Rij_2 - 1\frac{1}{2} \cdot Rij_1 \\ Rij_3 \to Rij_3 + \frac{1}{2} \cdot Rij_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 0 & 0 & 1 & \frac{\beta+1}{\alpha - 1\frac{1}{2}} \\ 0 & 1 & 0 & -1 - \frac{3\beta+1}{2\alpha-3} \\ 1 & 0 & 0 & 1 + \frac{\beta+1}{2\alpha-3} \end{array}\right)$$

Hieruit volgt: $(\alpha - 1\frac{1}{2})x_3 = \beta + 1$

$$x_3 = \frac{\beta + 1}{(\alpha - 1\frac{1}{2})}$$

Dus $Ax = b$ heeft geen oplossing voor $\alpha = 1\frac{1}{2}$. De β mag alles zijn.

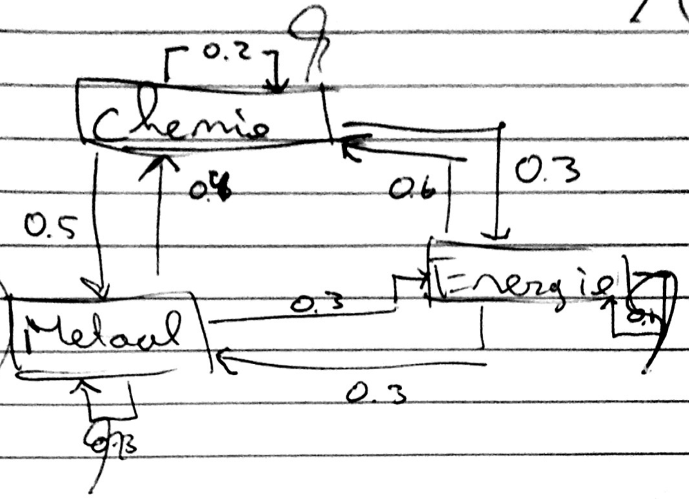
$\beta \neq 1$ en $\alpha = 3/2$ levert tegenstrijdigheid op

6. a ~~Chemie~~ Output from

3

Chemie	Energie	Metaal	Purchased By
0.2	0.6	0.4	Chemie
0.3	0.1	0.3	Energie
0.5	0.3	0.3	Metaal

b



3

c.

3

$$p_c = 0.2p_c + 0.6p_e + 0.4p_m$$
$$p_e = 0.3p_c + 0.1p_e + 0.3p_m$$
$$p_m = 0.5p_c + 0.3p_e + 0.3p_m$$

d

$$0.8p_c - 0.6p_e - 0.4p_m = 0$$
$$-0.3p_c + 0.9p_e - 0.3p_m = 0$$
$$-0.5p_c - 0.3p_e + 0.7p_m = 0$$

3

$$\left(\begin{array}{ccc|c} 0.8 & -0.6 & -0.4 & 0 \\ -0.3 & 0.9 & -0.3 & 0 \\ -0.5 & -0.3 & 0.7 & 0 \end{array}\right) \quad R_{y_1} \to R_{y_1} \cdot \frac{1}{0.8}$$

e.

$$\left(\begin{array}{ccc|c} 1 & -\frac{3}{4} & -\frac{1}{2} & 0 \\ -0.3 & 0.9 & -0.3 & 0 \\ -0.5 & -0.3 & 0.7 & 0 \end{array}\right) \quad \begin{array}{l} R_{y_2} \to R_{y_2} + 0.3 \cdot R_{y_1} \\ R_{y_3} \to R_{y_3} + 0.5 \cdot R_{y_1} \end{array}$$

0

$$\begin{array}{cccc} 1 & -\frac{3}{4} & -\frac{1}{2} & 0 \\ 0 & \frac{27}{40} & & \\ 0 & & & \end{array}$$

3a. $A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 5 & 6 \end{pmatrix}$

6

$A^2 = \begin{pmatrix} 7 & 12 & 0 & 0 & 0 \\ 18 & 21 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 89 & 104 \\ 0 & 0 & 0 & 65 & 76 \end{pmatrix}$

b. $A = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 5 & 6 \end{pmatrix}$ $\qquad A = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix}$

$A_{11}^{-1} = \begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} \cdot \frac{1}{5-6} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$

6

$A_{22} = \begin{pmatrix} A_{22_{11}} & 0 \\ 0 & A_{22_{22}} \end{pmatrix}$ $\qquad A_{22_{11}}^{-1} = \cancel{2}^{-1} = \frac{1}{2}$

$A_{22_{22}}^{-1} = \begin{pmatrix} 6 & -8 \\ -5 & 7 \end{pmatrix} = \frac{1}{2} = \begin{pmatrix} 3 & -4 \\ -2\frac{1}{2} & 3\frac{1}{2} \end{pmatrix}$

$A^{-1} = \begin{pmatrix} -5 & 2 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & -2\frac{1}{2} & 3\frac{1}{2} \end{pmatrix}$

4. $T(x_1, x_2, x_3) = (x_1 + x_2 + 3x_3,\ 3x_1 + x_2 + 5x_3,\ 2x_2 + 4x_3)$

8

$T(x) = (8, 16, 8)$

$x_1 + x_2 + 3x_3 = 8$

$3x_1 + x_2 + 5x_3 = 16$

$2x_2 + 4x_3 = 8$

In matrix vorm komt dit neer op:

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 3 & 1 & 5 & 16 \\ 0 & 2 & 4 & 8 \end{array}\right) \quad 1$$

$R\bar{y}_1 \to Ry_1$
$Ry_2 \to R\bar{y}_2 - 3Ry_1$
$Ry_3 \to Ry_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 3 & 8 \\ 0 & -2 & -4 & -8 \\ 0 & 2 & 4 & 8 \end{array}\right)$$

$Ry_1 \to Ry_1 + \frac{1}{2}Ry_2$
$Ry_2 \to Ry_2 \cdot -\frac{1}{2}$
$Ry_3 \to Ry_3 + Ry_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad 5$$

Oftewel:

$$\begin{aligned} x_1 + x_3 &= 4 \\ x_2 + 2x_3 &= 4 \\ 0 &= 0 \end{aligned} \quad \to \quad \begin{aligned} x_1 &= 4 - x_3 \\ x_2 &= 4 - 2x_3 \\ x_3 &\ \cancel{\text{is}}\ \text{vrij.} \end{aligned} \quad 2$$

$$\vec{x} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$ Voor alle waarden van x_3.

b. De transformatie is niet. onto ~~[illegible]~~ want er zijn
(2) meerdere oplossingen voor de ~~gegeven~~ opgave. ~~[illegible]~~
De ~~transformatie~~ is wel one to one er is ~~altijd een~~
~~waarde~~ voor elke $T(x)$ een uitkomst

5. a $T(x) = Ax$

(6) $A = B \cdot C$ $\quad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ & $C = \begin{pmatrix} \cos(-\frac{3}{4}\pi) & -\sin(-\frac{3}{4}\pi) \\ \sin(-\frac{3}{4}\pi) & \cos(-\frac{3}{4}\pi) \end{pmatrix}$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$$

Waar zijn B en C matrices van?

b. $T\begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$
(4)